

Comparative analysis of the optimization of error correcting codes and MIMO SVD schemes for increasing the energy efficiency of communications

Fernando Rosas*, Richard Demo Souza† and Christian Oberli‡

* Departement Elektrotechniek, Katholieke Universiteit Leuven, Belgium.

† CPGEI, Federal University of Technology Paraná, Brazil.

‡Departamento de Ingeniería Eléctrica, Pontificia Universidad Católica de Chile.

`fernando.rosas@esat.kuleuven.be`

Abstract

Developing techniques for increasing the energy efficiency of wireless communications is interesting for many technological, economical and ecological reasons. In this work we present a model for the energy consumption required for wireless data transfer. We show how the code rate of error correcting codes and multiple antenna schemes can be optimized from an energy-consumption point of view. We show interesting similarities between the optimal strategies, which relate energy efficiency and the multiplexing-diversity trade-off.

1 Introduction

The development of techniques for reducing the energy consumption of wireless communications is a central requirement for technologies like wireless sensor networks (WSN) to prosper into large-scale autonomous networks. The main tasks that the nodes of these networks perform are sensing the environment, processing the data and wireless communicating it. The latter task dominates the overall energy budget and, therefore, optimizing it has a direct impact on a network's lifetime [1]. Moreover, replacing batteries regularly is impractical in large networks or may even be impossible.

In this work we show how error control coding (ECC) and multiple-input multiple-output singular value decomposition (MIMO SVD) modulation can be optimized in order to increase the energy efficiency. ECC is a popular technique used for reducing the required transmit power by increasing the link reliability [2]. Nevertheless, while stronger codes provide better error correcting performance, usually they require more complex decoders with higher power consumption than simpler codes (c.f. [3]). Meanwhile, the MIMO SVD technique is an efficient method for sending data through a MIMO communications link [4]. The core concept considers the diagonalization of the channel matrix, thus establishing non interfering channels or “eigenchannels” [5]. Using all the eigenchannels maximizes the data rate, but sacrifices symbol error rate (SER). Conversely, using only some of them can yield a better SER but at the cost of decreasing the data-rate (c.f. [5,6]). Interestingly, the study of these two dissimilar scenarios leads to a similar solution: schemes that focus on diversity (low error rates) are optimal over long link distances, while schemes that focus on multiplexing (high data rates) are optimal for short-range links, suggesting an interesting and novel relationship between energy-efficiency with the diversity-multiplexing trade-off (c.f. [4]).

2 Energy consumption model

Our goal is to determine the total energy that is necessary for transferring one bit of data successfully in a point-to-point packet-switched MIMO communication. Such

a bit is called a “goodbit”. Following [7], every frame transmitted in the forward direction is matched by a feedback frame in the reverse direction that acknowledges correct reception or requests a retransmission. It is also assumed that all frames in both directions are detected and that all feedback frames are decoded without error.

In the sequel, Section 2.1 presents the analysis of the energy consumption of the transmitter of forward frames, which also decodes feedback frames. Section 2.2 then synthesizes the total energy consumption model. The analysis has been made for the MIMO transceiver architecture shown in Figure 1, which is frequently used among academic and commercial products (see e.g. [8]).

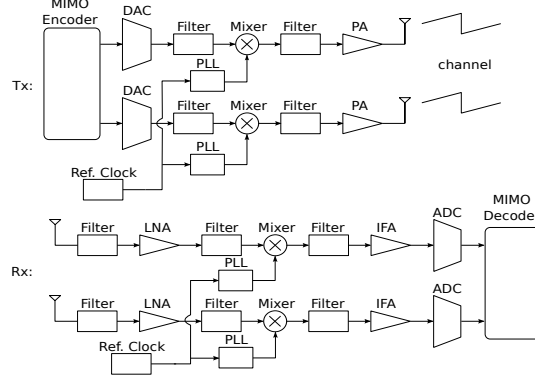


Figure 1: Common architecture of a MIMO transceiver.

2.1 Components of Energy Consumption of the Forward Transceiver

The energy consumed per goodbit by the transmitter is given by

$$\mathcal{E}_T = N_t \mathcal{E}_{st} + \mathcal{E}_{enc} + \left[\left(N_t P_{el,tx} + \sum_{k=1}^{N_t} P_{PA}^{(j)} \right) T_b + N_t P_{el,rx} \frac{T_{fb}}{L} \right] \tau . \quad (1)$$

Above, \mathcal{E}_{st} is the energy needed to wake up each transceiver branch of the transmitter from a low power consumption (sleep) mode, divided by the number of payload bits that are going to be transmitted before the transceiver goes again into low power consumption mode; \mathcal{E}_{enc} is the energy consumed by the baseband electronic processing required for encoding the forward frames; $P_{PA}^{(j)}$ is the power consumed by the power amplifiers (PAs) of the j -th transmission branch and $P_{el,tx}$ (respectively $P_{el,rx}$) is the total power per transceiver branch consumed by the remaining baseband and radio-frequency electronic components that perform the forward transmission (respectively the feedback frame reception); T_b is the average air time per payload bit on a forward frame, T_{fb} is the air time of the feedback frame and L is the number of payload bits per frame. It should be noticed that the startup and electronic consumption grows linearly with the number of antennas because the consumption of components that are common to all branches in the chosen architecture is negligible [9]. Finally, τ is the number of transmission trials until a frame is decoded without errors in the receiver. This quantity is a random variable whose distribution depends on PHY parameters as the SNR, channel statistics and modulation type, and on link layer parameters such as code rate, frame size and retransmission scheme (see Section 2.2.2).

Let us define $r = k/n$ as the code rate, where n is the number of bits per codeword and $n - k$ is the number of added redundancy bits. Then, each physical-layer forward frame carries H bits of header and a payload composed by rL bits of data and $(1 - r)L$

additional bits for coding. The total duration of a forward frame is shared by T_L seconds for transmitting the L bits of payload (with a suitable modulation), T_H seconds for the transmission of the header (with a binary modulation) and T_O seconds for the transmission of overhead signals for acquisition and tracking (channel estimation, synchronization, etc.). The average air time per data bit in a forward frame is

$$T_b = \frac{T_L + T_H + T_O}{rL} . \quad (2)$$

Let us define R_s as the physical layer symbol-rate of each transmitter antenna branch after the MIMO encoding and ω as the number of used eigenchannels (i.e. $1 \leq \omega \leq N_t$). If M_1, \dots, M_ω are the orders of the M -ary constellations used on the selected eigenchannels, then b is defined as the average $b = \omega^{-1} \sum_{k=1}^{\omega} \log_2 M_k$. Hence $R_b = \omega b R_s$ is the total bit rate of the MIMO system. By considering that header bits are sent using a binary modulation, and noting that $L/T_L = R_b$, then one can express T_b as

$$T_b = \frac{1}{rR_s} \left(\frac{1}{\omega b} + \frac{H}{\omega L} + \frac{N_t O_a + O_b}{L} \right) , \quad (3)$$

where O_a is the acquisition overhead per branch and O_b is the remaining overhead, which is approximately independent of the antenna array size. Both O_a and O_b are measured in bits. Analogously, one finds that

$$T_{fb} = \frac{F}{r\omega b R_s L} \quad (4)$$

is the feedback time per payload bit, where F is the number of bits per feedback frame.

Finally, the costs of encoding forward frames, which is shared among the rL data bits, are due to the computations required for implementing the ECC and MIMO SVD modulation. This can be calculated as

$$\mathcal{E}_{\text{enc}} = \mathcal{E}_{\text{enc}}^{\text{ECC}}(r) + \mathcal{E}_{\text{enc}}^{\text{SVD}}(\omega) = \frac{V_{\text{dd}} I_0}{rL f_{\text{APU}}} \sum_{j=1}^{J_{\text{APU}}} c_j \left[n_j^{\text{enc,ECC}}(r) + n_j^{\text{enc,SVD}}(\omega) \right] . \quad (5)$$

Above, V_{dd} is the arithmetic processing unit (APU) operating voltage and I_0 is the average current during the execution time of the arithmetic operations, f_{APU} is the APU's clock frequency, c_j is the number of clock cycles required by the j -th operation which is performed $n_j^{\text{enc,ECC}}$ (resp. $n_j^{\text{enc,SVD}}$) times during the ECC algorithm (resp. MIMO SVD algorithm) and J_{APU} is the number of different operations the APU performs.

2.2 Total Energy per Successfully Transferred Bit

In analogy with (1), the total energy used by the receiver for demodulating τ forward transmissions and for transmitting the corresponding τ feedback frames is

$$\mathcal{E}_R = N_r \mathcal{E}_{\text{st}} + \left[\mathcal{E}_{\text{dec}} + N_r P_{\text{el,rx}} T_b + \left(N_r P_{\text{el,tx}} + \sum_{k=1}^{N_r} P_{\text{PA}}^{(j)} \right) \frac{T_{fb}}{L} \right] \tau , \quad (6)$$

where \mathcal{E}_{dec} accounts for the energy consumption of decoding the forward frame, which is defined using (5) with obvious modifications. Note that this term is multiplied by τ because a new decoding algorithm has to be performed for each transmission trial.

Let us define the total energy per goodbit as $\mathcal{E}_b = \mathcal{E}_T + \mathcal{E}_R$. Because of τ , \mathcal{E}_b is a random variable that depends on the realizations of the channel and the thermal noise. In order to simplify further calculations let us assume $N_t = N_r = N$ (the extension of the presented results to the non-symmetrical case is straightforward). Then, the mean energy consumption per bit is given by

$$\bar{\mathcal{E}}_b = \mathbb{E} \{ \mathcal{E}_T + \mathcal{E}_R \} = NS + \mathcal{E}_{\text{enc}} + \left[\mathcal{E}_{\text{dec}} + \left(NP_{\text{el}} + \sum_{k=1}^N P_{\text{PA}}^{(j)} \right) T \right] \bar{\tau} , \quad (7)$$

where $S = 2\mathcal{E}_{\text{st}}$ is the total startup energy per transceiver branch, $P_{\text{el}} = P_{\text{el,tx}} + P_{\text{el,rx}}$ is the total power consumed by electronic components per branch and $T = T_b + T_{\text{fb}}/L$ is the total time per bit. In the remaining of this section we will seek for explicit expressions for the dependence of $P_{\text{PA}}^{(j)}$ and $\bar{\tau}$ on the SNR.

2.2.1 PA's total power consumption as function of the SNR

Let us define $P_A^{(j)}$ as the power radiated by the antenna of the j -th branch, which is supplied by a corresponding PA (Figure 1). This quantity can be modeled as $P_A^{(j)} = (\eta/\xi)P_{\text{PA}}^{(j)}$, where η is the drain efficiency of the PA and ξ is the peak-to-average ratio of the transmitted signal [10]. Using the result from Appendix A of [5] we have that

$$\sum_{j=1}^N P_{\text{PA}}^{(j)} = \frac{\xi}{\eta} \sum_{k=1}^{\omega} \bar{P}_{\text{tx}}^{(k)} , \text{ sy} \quad (8)$$

where $\bar{P}_{\text{tx}}^{(k)}$ is the transmission power that has been allocated to the k -th eigenchannel (this result simply states that the irradiated energy is the same if is counted over antennas or eigenchannels). The irradiated power attenuates over the air with path loss and arrives at the receiver with a mean power given by $\bar{P}_{\text{rx}}^{(k)} = \frac{\bar{P}_{\text{tx}}^{(k)}}{A_0 d^\alpha}$, where A_0 is a parameter that depends on the transmitter and receiver antenna gains and the transmission wavelength, d is the distance between transmitter and receiver and α is the path loss exponent.

Let us define $\bar{\gamma} = \omega^{-1} \left(\sum_{k=1}^{\omega} \bar{P}_{\text{rx}}^{(k)} / \sigma_n^2 \right)$ as the average SNR among the used eigenchannels at the decision stage, where σ_n^2 is the noise power. In general, $\sigma_n^2 = N_0 W N_f M_L$, where N_0 is the power spectral density of the baseband-equivalent additive white Gaussian noise (AWGN), W is the transmission bandwidth, N_f is the noise figure and M_L is a link margin term which represents any other additive noise or interference [11]. Finally, the total power consumption of the PAs can be written as

$$\sum_{j=1}^N P_{\text{PA}}^{(j)} = \frac{\xi A_0 d^\alpha}{\eta} \sum_{k=1}^{\omega} \bar{P}_{\text{rx}}^{(k)} = \frac{\xi A_0 \sigma_n^2}{\eta} d^\alpha \omega \bar{\gamma} = A d^\alpha \omega \bar{\gamma} . \quad (9)$$

2.2.2 $\bar{\tau}$ as function of the SNR

A key contributor to the energy consumption is the need for re-transmissions due to forward frames that get decoded with errors. The number of trials, τ , until a frame is correctly decoded is a random variable, whose mean value has been shown to be [7]

$$\bar{\tau} = 1 + \sum_{j=1}^{\infty} \mathbb{E} \left\{ \prod_{i=1}^j P_{\text{f}}(i) \right\} , \quad (10)$$

where $\mathbb{E}\{\cdot\}$ denotes the expectation operator and $P_f(i)$ is the probability of decoding the frame with error during the i -th transmission trial. In general, the $P_f(i)$ are random variables that depend on the frame size, modulation type and received SNR during the i -th trial. It is to be noted that (10) is a general formula which is valid for correlated or uncorrelated channel fading statistics [7].

Let us assume now that the probabilities of frame error of each transmission trial, $\{P_f(i)\}_{i=1}^\infty$ are a set of i.i.d. random variables. It can be shown that τ is a geometric random variable with parameter $\bar{P}_f := \mathbb{E}\{P_f(i)\}$, i.e. $\mathbb{P}\{\tau = r\} = (1 - \bar{P}_f) \bar{P}_f^{r-1}$. Its mean value can be found by direct calculation, or using (10) as follows:

$$\bar{\tau} = 1 + \sum_{j=1}^{\infty} (\bar{P}_f)^j = \frac{1}{\bar{P}_f^*}, \quad (11)$$

where $\bar{P}_f^* = 1 - \bar{P}_f$ is the probability of decoding the frame correctly. In the following we derive expressions for \bar{P}_f^* for coded single antenna and uncoded MIMO SVD transmissions. Expressions for coded MIMO transmissions are ongoing work.

- **Uncoded MIMO SVD systems:** The value of the mean frame error rate depends on how the data symbols are fed into the SVD engine. In particular, it has been shown that a pseudo-random feeding of the encoder outperforms an ordered feeding [5]. Hence, we will consider the case in which the bits of each frame are assigned to the ω used eigenchannels following a different order for each transmission trial in a pseudo-random fashion. For simplicity we will assume that all eigenchannels are used with M -ary modulations of equal size and that each used eigenchannel uses an equal amount of irradiation power (the case of different modulation sizes and irradiation power is addressed in [6]). Under those assumptions, it can be shown that [5]

$$\bar{\tau}_{\text{SVD}} = \left[1 - \frac{1}{\omega} \sum_{k=1}^{\omega} \bar{P}_{\text{bin}}(\lambda_k \bar{\gamma}) \right]^{-H} \left[1 - \frac{1}{\omega} \sum_{k=1}^{\omega} \bar{P}_s(\lambda_k \bar{\gamma}) \right]^{-l}, \quad (12)$$

where \bar{P}_{bin} and \bar{P}_s are the BPSK and M -ary modulation mean symbol error rate and $l = L/\log_2(M)$ is the number of M -ary symbols per frame.

- **Single antenna systems with linear block codes:** For $n < L$ let's define $n_c = L/n$ ($n_c \in \mathbb{N}$) as the number of codewords per payload. Then, to decode a frame correctly one needs H correct header symbols and n_c codewords with at least $(n - t) = \lambda$ correct symbols, where t is the maximum number of bits that the FEC block code is able to correct per codeword. Hence, by taking into account the various permutations, \bar{P}_f^* can be written as

$$\bar{\tau}_{\text{ECC}} = [1 - \bar{P}_{\text{bin}}(\bar{\gamma})]^{-H} \left[\sum_{j=0}^t \binom{n}{j} [1 - \bar{P}_b(\bar{\gamma})]^{n-j} \bar{P}_b(\bar{\gamma})^j \right]^{-n_c}, \quad (13)$$

where \bar{P}_b is the mean bit error rate of the M -ary modulation.

3 Minimizing the energy per bit

In this section we analyze how the energy consumption of wireless communications depends on the SNR (Section 3.1), the code rate of ECC and the number of used eigenchannels of MIMO SVD (Section 3.2).

3.1 Optimization of the SNR

The mean total energy consumption per goodbit (7) can be rewritten using (9) and (11), so that the dependence on the SNR ($\bar{\gamma}$), number of used eigenchannels (ω) and antenna array size (N) becomes explicit. Concretely:

$$\bar{\mathcal{E}}_b(\bar{\gamma}, \omega, r) = NS + \mathcal{E}_{\text{enc}}(\omega, r) + \frac{\mathcal{E}_{\text{dec}}(\omega, r) + [NP_{\text{el}} + Ad^\alpha \omega \bar{\gamma}] T(\omega, r)}{1 - \bar{P}_f(\bar{\gamma}, \omega, r)} . \quad (14)$$

Just by considering the structure of (14) it can be seen that the energy consumption is large at extreme values of the total SNR. In effect, if the SNR is low then the frame error rate tends to one, and hence the energy consumption is high because of the large number of retransmissions needed for a successful frame reception. On the contrary, at a high SNR the right-hand term of (14) is also large because the radiated power (which is proportional to $\bar{\gamma}$) is excessive. We thus infer that an optimal SNR that minimizes the energy consumption must exist in between. In fact, for typical fading channel models (e.g. Rayleigh or Nakagami- m models) (14) will be a convex function of the SNR, having a unique minimum. Let us define the optimal SNR at which the system attains a minimal average energy consumption as

$$\bar{\gamma}_{\omega, r}^* = \underset{\bar{\gamma} \in [0, \infty)}{\operatorname{argmin}} \bar{\mathcal{E}}_b(\bar{\gamma}, \omega, r) . \quad (15)$$

Therefore, $\bar{\gamma}_{\omega, r}^*$ represents an optimal trade-off between radiation power and consumption because of retransmissions.

Let us consider now a family of BCH codes with fixed codeword length n and varying information content k , where each code can be parameterized by its code rate $r = k/n$. Let us denote the set of all the BCH codes for a given value of n as \mathcal{R}_n (the complete list of BCH codes for various values of n can be found in [2]). Then, for a given codeword length n and antenna array size N , the optimal code rate and multiplexing gain is given by

$$(\omega^*, r^*) = \underset{\substack{\omega \in \{1, \dots, N\} \\ r \in \mathcal{R}_n}}{\operatorname{argmin}} \bar{\mathcal{E}}_b(\bar{\gamma}_{\omega, r}^*, \omega, r) . \quad (16)$$

Note that in the above definition the consumption is evaluated when the system is using its own optimal mean SNR $\bar{\gamma}_{\omega, r}^*$, as defined in (15).

3.2 Compared results of ECC and MIMO SVD

In the following we compare how ω^* and r^* change as function of the link distance for two cases: single antenna systems using BCH codes, for which $\omega = 1$ and r is variable, and uncoded MIMO SVD transmissions, for which $r = 1$ and $\omega \in \{1, 2, \dots, N\}$. Figures 2a and 2b show evaluations of (14) as function of SNR using parameters of typical low-power devices (c.f. [3, 5]).

For the case of single antenna systems using BCH codes it has been show [3] that r^* increases as the transmission distance decreases (see Figure 2e). Although the rate of change depends on the channel fading statistics, the qualitative scenario is always the same: for long transmission distances it is optimal to use a significant amount of coding (small r), while for short distances it is best to use almost no coding at all (large r). Both results agree with intuition. In effect, for long-range communications the power consumed by the power amplifier (equal to $Ad^\alpha \bar{\gamma}^*$ Watts) dominates over the consumption of other components. Hence, it is useful to use coding in order to reduce the SNR requirements for attaining a given SER. On the contrary, for short

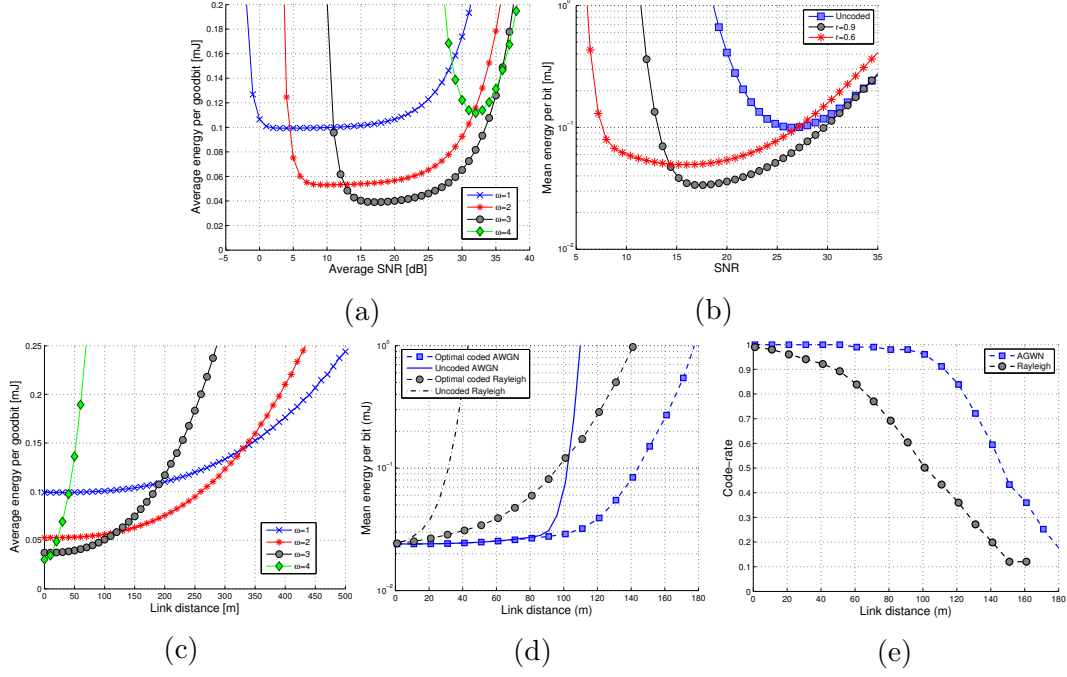


Figure 2: (a) Energy consumption of a 4×4 MIMO SVD system for various possibilities of used eigenchannels and a link distance of $d = 45$ meters, (b) energy consumption of transmissions using BCH coding and various code rates with $d = 50$ meters, (c) minimal energy consumption of a 4×4 MIMO SVD system that uses uncoded BPSK with equal power allocation, (d) minimal energy consumption of transmissions which use BCH codes with optimal code rate and (e) optimal code rate of BCH codes as function of link distance.

link distances it is found that $\bar{\mathcal{E}}_b \approx S + \mathcal{E}_{\text{enc}} + (\mathcal{E}_{\text{dec}} + P_{\text{el}}T)\bar{\tau}$, and therefore it becomes attractive to use less redundancy in order to reduce the total time per bit and the baseband consumption of encoding and decoding.

When optimizing uncoded MIMO SVD transmissions one finds results that resembles the case of ECC. In effect, beamforming ($\omega = 1$) is the energy-optimal transmission scheme for long range communications, because it invests the radiated electromagnetic energy entirely on the best eigenchannel [5]. This reduces the mean SER and thereby limits the average number of retransmissions. For medium link distances (below 350 meters in Figure 2c) the power consumed by electronic components begins to dominate over the radiated power and therefore also over the consumption of the power amplifiers. Therefore, it is attractive to use more eigenchannels simultaneously, because it increases the baud rate and thereby reduces the transmission time per bit. At very short link distances ($d \leq 10$ meters in Figure 2c), this notion leads to using full SVD ($\omega = N$) as the most energy-efficient scheme.

These results make us think in terms of the well known multiplexing-diversity trade-off [12]. Although this trade-off was formulated for MIMO systems, it can be stated in more general terms as *communications resources can be invested to achieve many bad-quality bits (high throughput with high error rates) or few low-quality bits (low throughput with low error rates)*. For the case of ECC, high code rates can be related with diversity and low ones with multiplexing; in the same fashion for MIMO SVD systems, a large number of eigenchannels provides multiplexing capabilities while beamforming achieves diversity. This interpretation unifies the results presented in this work, and suggest the following general principle: *energy-efficient long range transmissions are achieved using transmission schemes that focus on diversity, while energy-efficient short-range communications are achieved using schemes that focus on multiplexing.*

4 Conclusions

We studied the effects of ECC and MIMO SVD on the energy efficiency of wireless communications. We considered the energy cost of electronic components, of the base-band processing, and the effects of retransmissions. For achieving energy efficiency, results suggest that transmission schemes which focus on diversity are best for long range communications and schemes which focus on multiplexing are better for short distances. In the case of ECC this means that low code rates are optimal for long transmission distances while the optimal code rate increases as the transmission distance shortens. Analogously, the optimal number of used eigenchannels in a MIMO SVD link is small for long distances, while it is larger for short distances. Hence, the link distance is a critical parameter for designing the energy-optimal combination of multiplexing and diversity features of a communication system.

References

- [1] A. W. Holger Karl, *Protocols and Architectures for Wireless Sensor Networks*. John Wiley & Sons inc., 2005.
- [2] S. Lin and D. J. Costello, *Error Control Coding: Fundamentals and Applications*, 2nd ed. Prentice-Hall, June 2004.
- [3] F. Rosas, G. Brante, R. Souza, and C. Oberli, “Optimizing the code rate for achieving energy-efficient wireless communications,” in *Proc. WCNC’14*, accepted for publication, 2014.
- [4] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*. Cambridge University Press, 2005.
- [5] F. Rosas and C. Oberli, “Energy-efficient MIMO SVD communications,” in *Proc. IEEE PIMRC’12*, Sep. 2012.
- [6] —, “Impact of transmitter-side CSI on the energy-efficiency of MIMO communications,” *IEEE Trans. Wireless Commun.*, under revision.
- [7] —, “Modulation and SNR optimization for achieving energy-efficient communications over short-range fading channels,” *IEEE Transactions on Wireless Communications*, vol. 11, no. 12, pp. 4286–4295, 2012.
- [8] K. Nishimori, R. Kudo, N. Honma, Y. Takatori, and M. Mizoguchi, “16x16 Multiuser MIMO Testbed Employing Simple Adaptive Modulation Scheme,” in *Proc. VTC Spring 2009*, Apr. 2009, pp. 1–5.
- [9] S. Cui, A. J. Goldsmith, and A. Bahai, “Energy-constrained modulation optimization,” *IEEE Transactions on Wireless Communications*, vol. 4, no. 5, pp. 2349–2360, 2005.
- [10] T. H. Lee, *The Design of CMOS Radio-Frequency Integrated Circuits*. Cambridge University Press, 1998.
- [11] S. Cui, A. J. Goldsmith, and A. Bahai, “Energy-efficiency of MIMO and cooperative MIMO techniques in sensor networks,” *IEEE Journal on Selected Areas in Communications*, vol. 22, no. 6, pp. 1089–1098, Aug. 2004.
- [12] L. Zheng and D. N. C. Tse, “Diversity and multiplexing: a fundamental tradeoff in multiple-antenna channels,” *IEEE Transactions on Information Theory*, vol. 49, no. 5, pp. 1073–1096, May 2003.